8th International Congress of Croatian Society of Mechanics
29 September – 2 October 2015
Opatija, Croatia

Dynamic Sensitivity of a Multi-block Assembly Subjected to Horizontal Harmonic Excitation

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Abstract. Dynamic sensitivity of a one-dimensional assembly of eight rigid blocks undergoing external harmonic vibrations is investigated. Several parameters are considered as dynamic attributes: time variation of the mass inertia of the assembly and for each block the average over all the simulation of granular temperature, relative velocity, coordination number. Numerical simulation is based on the Non Smooth Contact Dynamics (NSCD) time integration framework Solfec (http://code.google.com/p/solfec/). Sensitivity space parameters include a range of excitation frequencies and velocity amplitudes.

1 Introduction

Many civil engineering structures can be considered as discrete, discontinuous systems with deliberate gaps or clearances ranging from simple dry stone walling or masonry to sophisticated complex geometry graphite cores in nuclear power plants. In spite of extraordinary advances in nonlinear computational mechanics, it is still difficult to assess or characterise the highly nonlinear mechanical response of such systems even if some homogenisation technique allows relevant dynamic characterisation. The way we chose to explore and illustrate relevant observations is to study a one-dimensional model *i.e.* a row comprising N rigid blocks driven by harmonic excitation of the side boundaries, where the blocks are subject to dissipative collisions both among themselves and the side boundary.

Here we study the behaviour of an assembly of eight rigid blocks subject to external harmonic excitation due to collisions with a rigid boundary undergoing a prescribed velocity history. We want to see if clustered states or fluidized regime appear as it has been noticed in other experiments and simulations [1,2]. Indeed these phenomena depend on the local dissipative mechanical interactions, on the initial configuration of the assembly and also on the frequencies and amplitude of the external excitation. Moreover, the intention is to explore possible self organisation of the clustered blocks into oscillatory patterns, associated with distinct frequencies. The control parameters for the study are frequencies and velocity amplitude of the boundary.
Details of the numerical model adopted are given first, followed by the study of the influence of varying excitation frequencies and velocity amplitudes.

2 Numerical model

2.1 System and simulation

A system of eight identical rigid blocks is considered. With the block sizes $s = 40$ mm and the section $10 * 113\ mm^2$ (mass density $1200\ kg/m^3$, hence block masses equal to $54.24\ g$) the blocks are initially in contact with each other within a horizontal box of width $114\ mm$ and a length of $394\ mm$ (which implies a free space of $74\ mm$ in the cell). The first block is initially in contact with the side boundary of the cell. The cell is driven horizontally by a series of sinusoidal motions

$$U_{cell}(t) = Asin(\omega t)$$

with amplitude $A = 7\ mm$ or $A = 15\ mm$, with frequencies $f = (0.1; 0.5; 1; 2; 2.5; 3; 3.5; 4)$ Hz (angular frequencies $\omega = 2\pi f = (0.62; 3.14; 6.28; 12.56; 15.70; 18.84; 21.99; 25.13)$ rad/s) hence the boundary velocity V_{cell} at the time t is defined as

$$V_{cell}(t) = A\omega cos(\omega t)$$

A schematic drawing is given in Figure 1. The friction between blocks and with the cell boundary is taken as $\mu = 0.03$ and the restitution coefficient e for a collision between blocks and between the block and the cell boundary are $e = 0.3$.

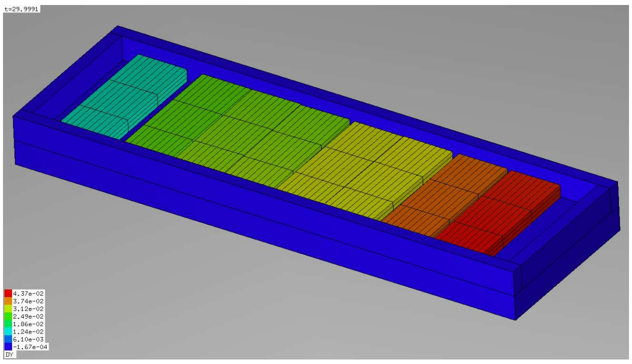
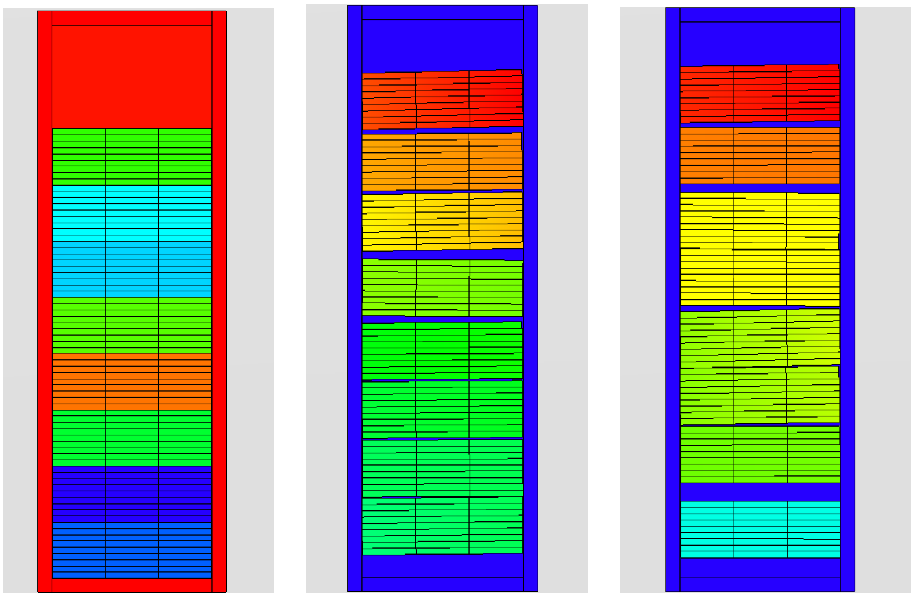


Figure 1: Solfec numerical model



(a) (b) (c)

Figure 2: Block assembly at (a) $t = 0$ s, (b) $t = 10$ s, and (c) $t = 30$ s.

Discrete numerical simulations were performed using the Non Smooth Contact Dynamics (NSCD) method [5], which is specially convenient for rigid blocks. This method is based on an implicit time integration of the equations of motion expressed in terms of velocities and considers generalized nonsmooth contact laws describing noninterpenetration and dry friction between rigid blocks. This formulation unifies the description of lasting contacts and collisions through the concept of an impulse, which can be defined as the time integral of the contact force. The NSCD simulation was applied using the Solfec platform [4].

2.2 Dynamic sensitivity parameters

Dynamic sensitivity of the eight block assembly under harmonic excitation is studied by extracting time histories of several scalar parameters from the simulation results − the mass inertia, the agitation, maximum relative velocity between the blocks and the coordination number of each block, which are conveniently captured from the NSCD/DEM method [2,3].

First trajectories of both the left and the right sides of each block as well as the cell boundary are traced, allowing visualisation of shocks between the blocks. Then, for a series of simulations with differing excitation frequencies, maximum relative velocity between the blocks, coordination number of each block and agitation θ achieved by each block of the system (that is block i of mass m_i, of position $Y_i(t)$ and velocity $V_i(t)$) are recorded over the entire simulation time T for a given boundary velocity amplitude and a given angular frequency. The agitation [6] is defined as follows:

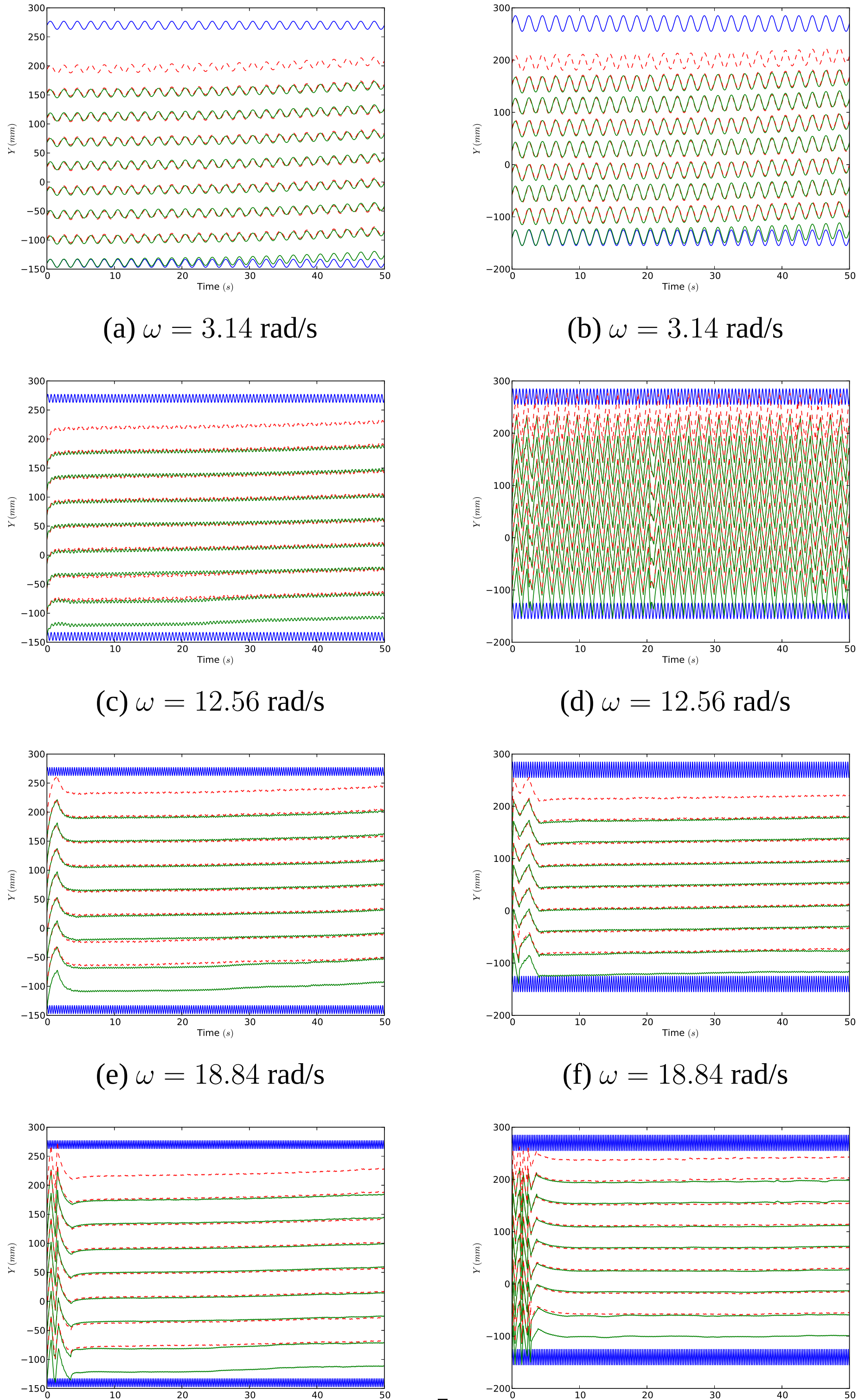
$$\theta = \frac{1}{2} m_i (\langle V_i^2 \rangle - \langle V_i \rangle^2)$$

where $\langle V_i \rangle = \frac{1}{T} \sum_0^T V_i$ and $\langle V_i^2 \rangle = \frac{1}{T} \sum_0^T V_i^2$, respectively the mean velocity and the mean square velocity of the assembly over the entire simulation. The agitation parameter θ corresponds to the fluctuative part of the particle motion and it is related to granular temperature [7]. In addition, we plotted the mass inertia index $I_z(t)/I_{ref}$ where $I_z(t)$ is the maximum recorded mass inertia during the excitation, extracted with the mass inertia, defined as follows:

$$I_z(t) = \sum_i m_i (Y_i(t) - Y_{cm}(t))^2$$

and normalised with respect to the minimum mass inertia I_{ref} the block group has when they are all packed together. The trajectory of the center of mass is $Y_{cm}(t) = \frac{1}{8} \sum_{i=1}^{8} Y_i(t)$. Finally oscillatory behaviour of the assembly can be illustrated with two phase plane diagrams: comprising the mass inertia index I_z/I_{ref} *vs* the time rate of change of the mass inertia index $\frac{d(I_z/I_{ref})}{dt}$; and comprising the velocity of the block system centroid $V_{cm}(t) = \frac{1}{8} \sum_{i=1}^{8} V_i(t)$ *vs* the trajectory of the center of mass of the assembly $Y_{cm}(t)$.

3 Time evolution of the blocks distribution



(a) $\omega = 3.14$ rad/s

(b) $\omega = 3.14$ rad/s

(c) $\omega = 12.56$ rad/s

(d) $\omega = 12.56$ rad/s

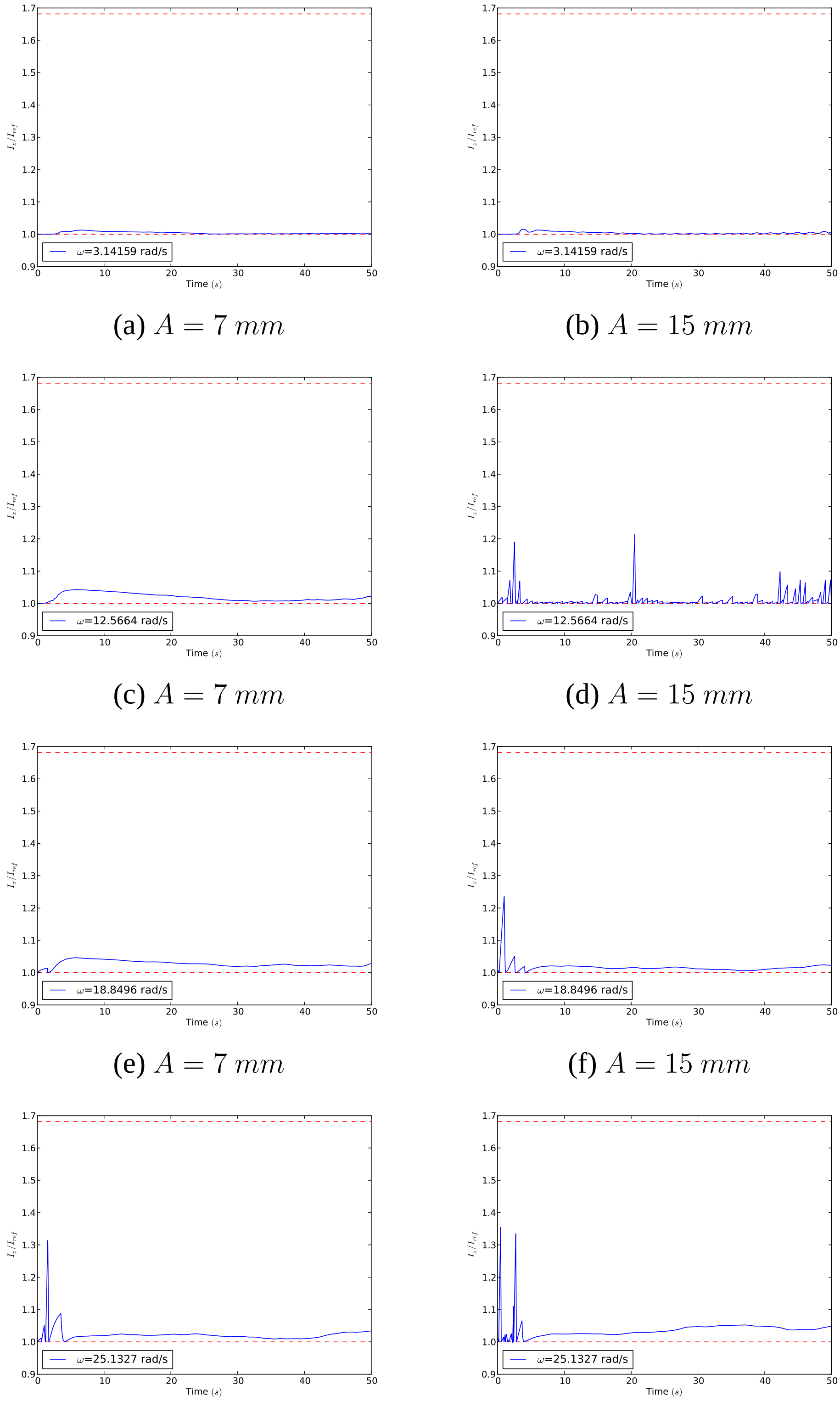
(e) $\omega = 18.84$ rad/s

(f) $\omega = 18.84$ rad/s

(g) $\omega = 25.13$ rad/s

(h) $\omega = 25.13$ rad/s

Figure 3: Trajectories time evolution Y of the left and the right sides of each block and cell for constant displacement amplitude $A = 7\ mm$ (a), (c), (e), (g) and $A = 15\ mm$ (b), (d), (f) and (h).



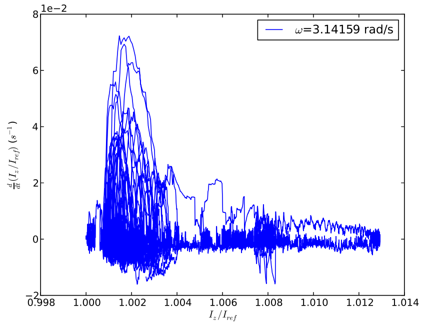
(a) $A = 7\ mm$ (b) $A = 15\ mm$

(c) $A = 7\ mm$ (d) $A = 15\ mm$

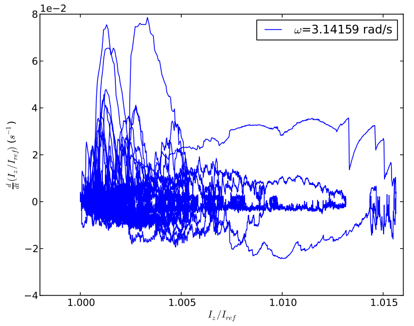
(e) $A = 7\ mm$ (f) $A = 15\ mm$

(g) $A = 7\ mm$ (h) $A = 15\ mm$

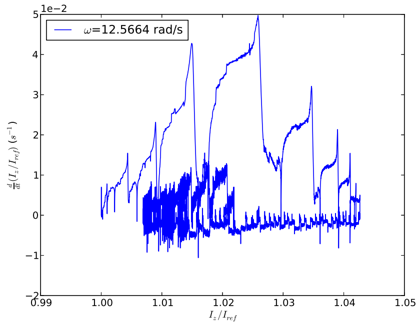
Figure 4: Index $I_z(t)/I_{ref}$ time evolution of the stack for constant displacement amplitude A. Index can vary between the lower value I_{ref} and the bigger when the blocks are spread all over the cell so between 1 and ≈ 1.68.



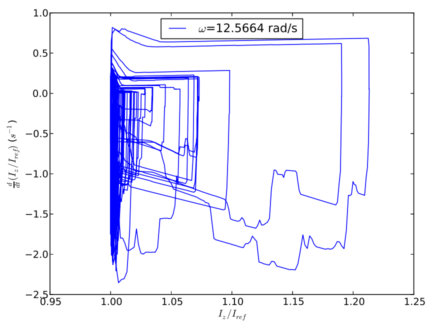
(a) $A = 7\ mm$



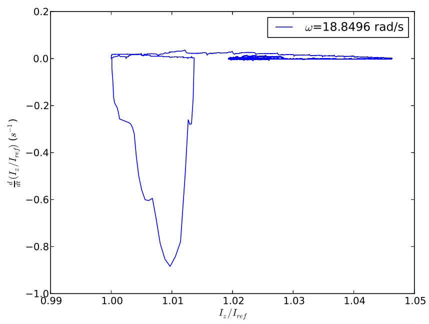
(b) $A = 15\ mm$



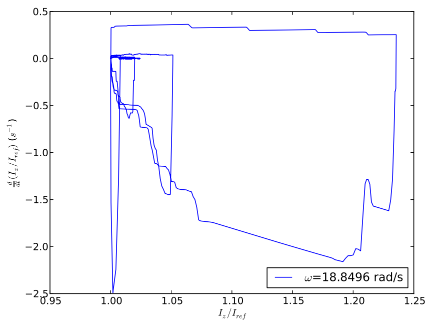
(c) $A = 7\ mm$



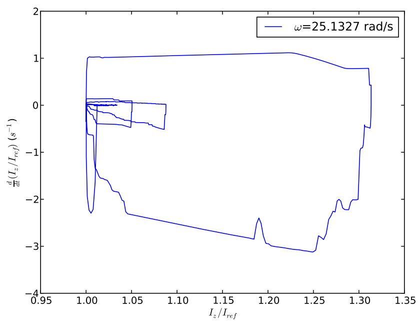
(d) $A = 15\ mm$



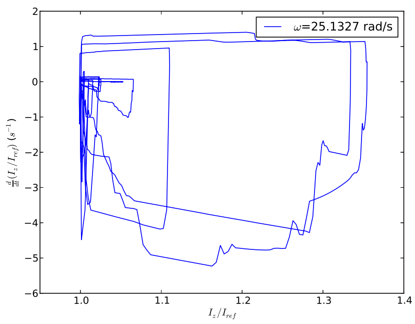
(e) $A = 7\ mm$



(f) $A = 15\ mm$



(g) $A = 7\ mm$



(h) $A = 15\ mm$

Figure 5: Phase plane diagram of the mass inertia index of the block assembly for constant displacement amplitude A.

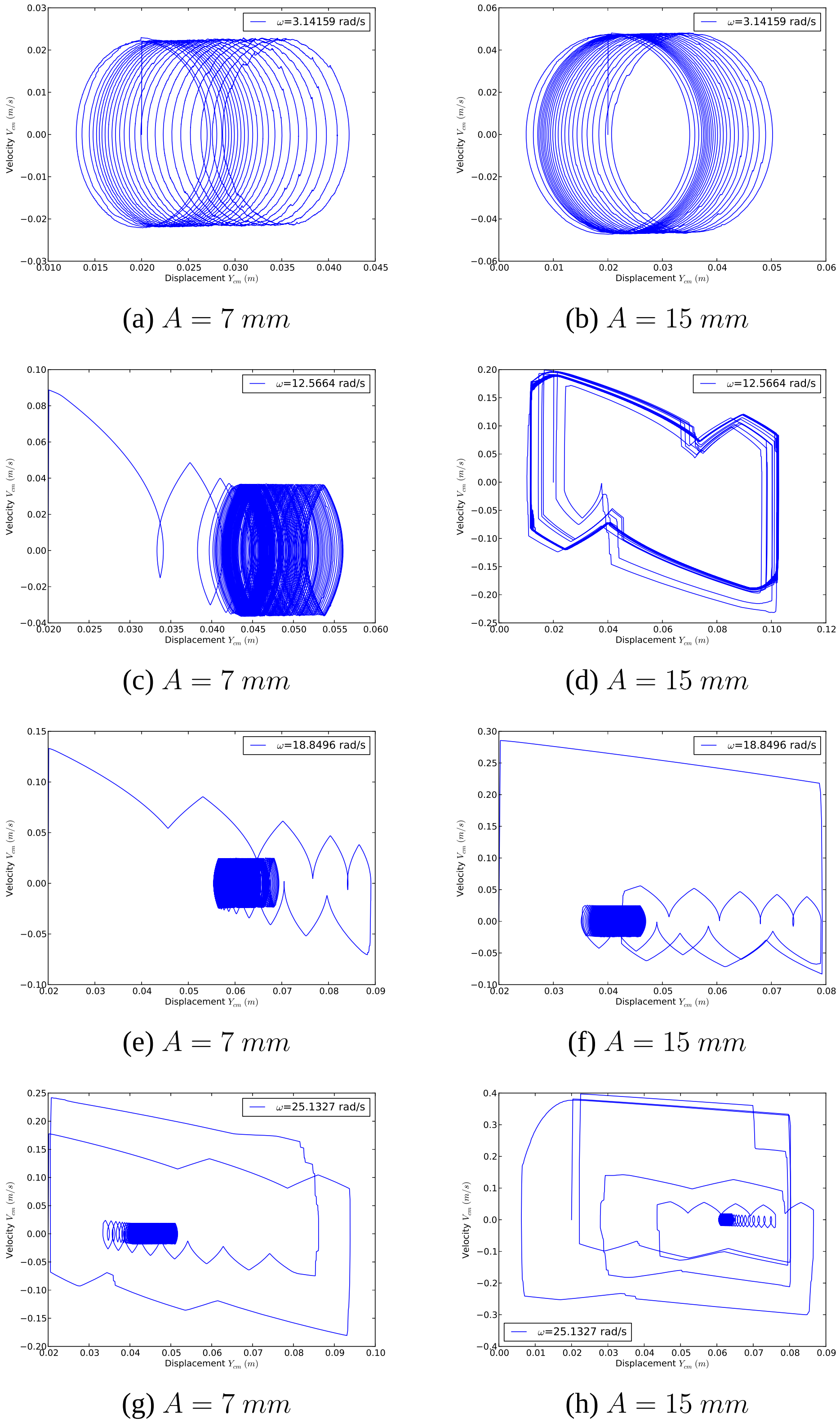
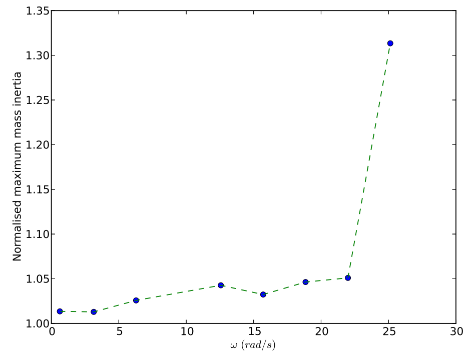
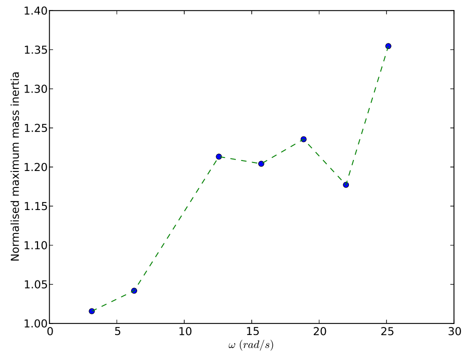


Figure 6: Phase plane diagram of the position of the centroid of the block assembly for constant displacement amplitude A.

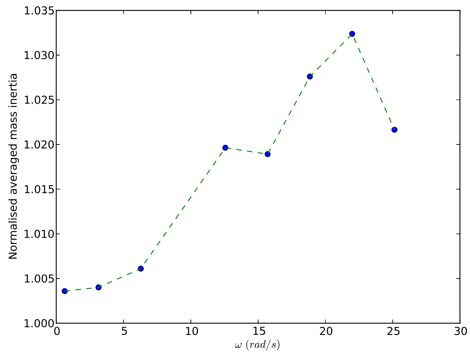
4 Averaged characteristics over the entire simulation



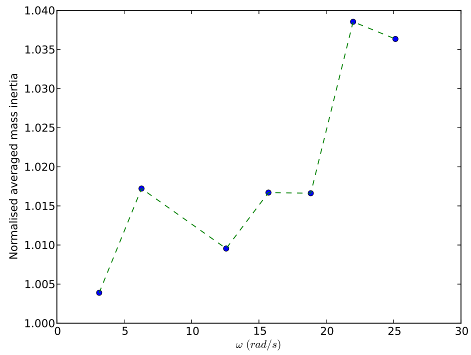
(a) $A = 7\ mm$



(b) $A = 15\ mm$

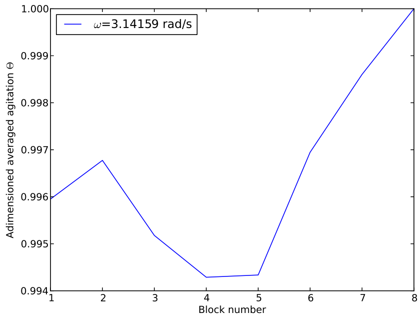


(c) $A = 7\ mm$

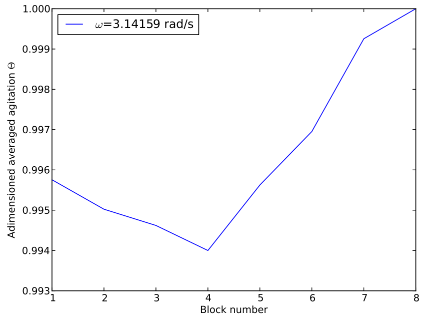


(d) $A = 15\ mm$

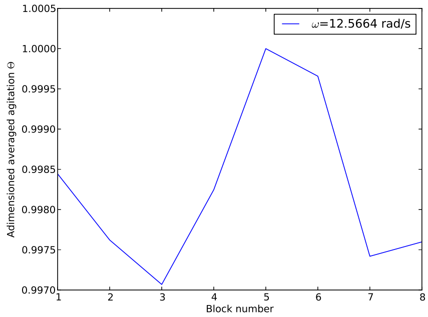
Figure 7: (a) and (b) maximum index $I_z(t)/I_{ref}$ acheived over the entire simulation according to ω for constant displacement amplitude A. (c) and (d) averaged index $I_z(t)/I_{ref}$ computed all over the time simulation according to ω for constant displacement amplitude A.



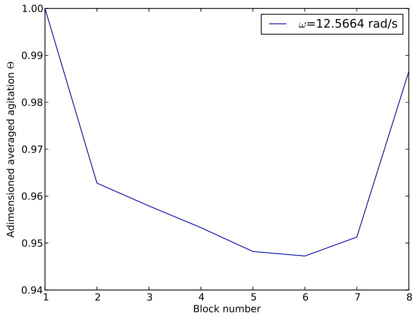
(a) $A = 7\ mm$



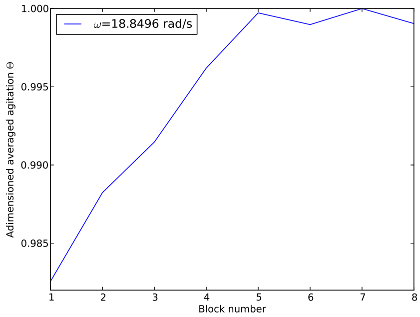
(b) $A = 15\ mm$



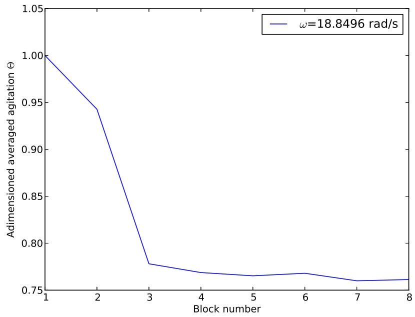
(c) $A = 7\ mm$



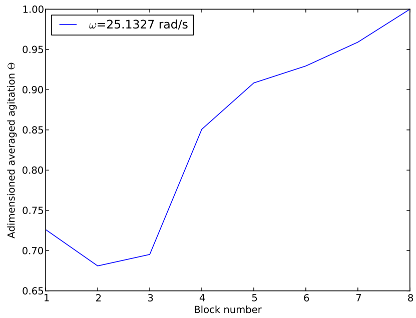
(d) $A = 15\ mm$



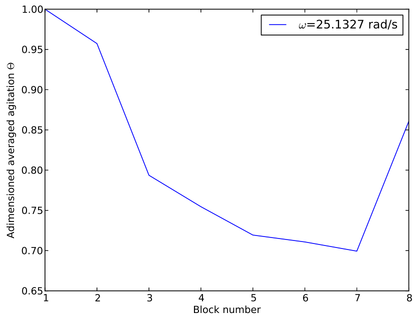
(e) $A = 7\ mm$



(f) $A = 15\ mm$

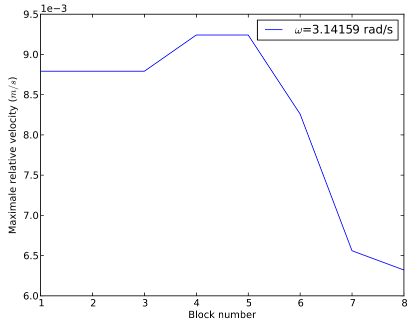


(g) $A = 7\ mm$

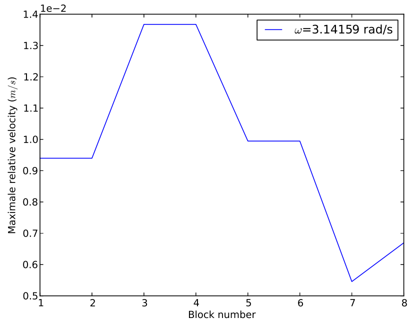


(h) $A = 15\ mm$

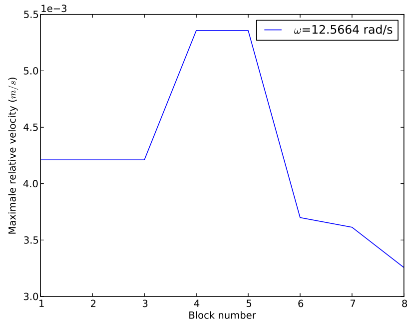
Figure 8: Adimensioned agitation θ of each block of the assembly for constant displacement amplitude A.



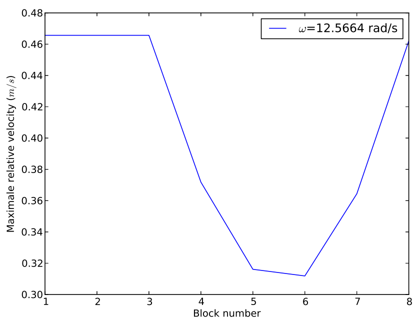
(a) $A = 7\ mm$



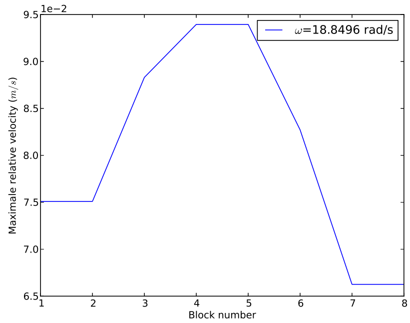
(b) $A = 15\ mm$



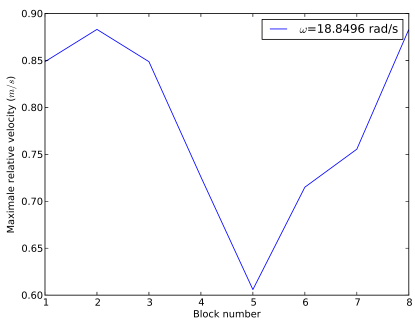
(c) $A = 7\ mm$



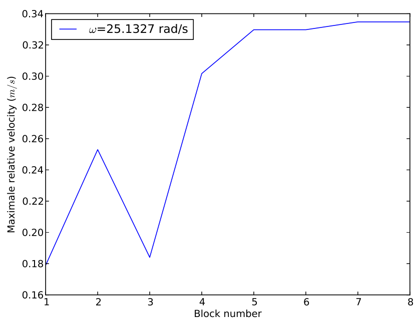
(d) $A = 15\ mm$



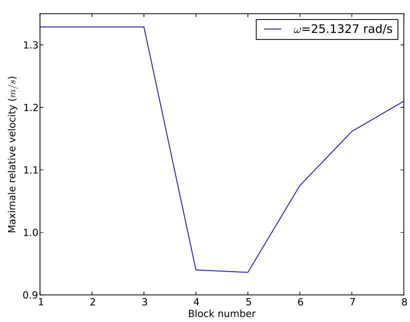
(e) $A = 7\ mm$



(f) $A = 15\ mm$

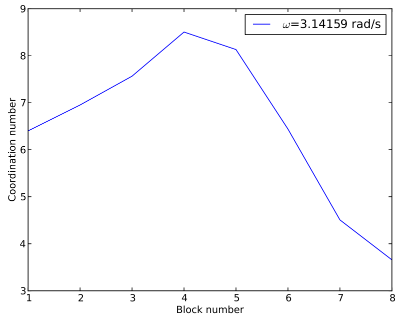


(g) $A = 7\ mm$

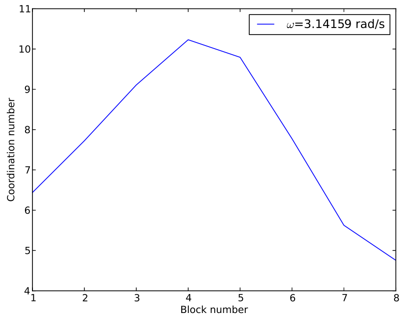


(h) $A = 15\ mm$

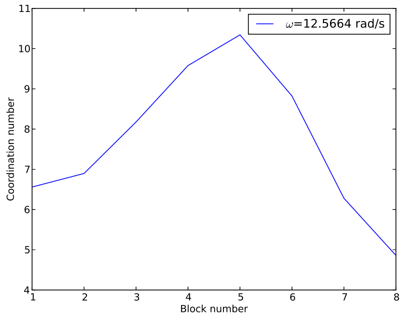
Figure 9: Maximum relative velocity between the blocks of the assembly for constant displacement amplitude A.



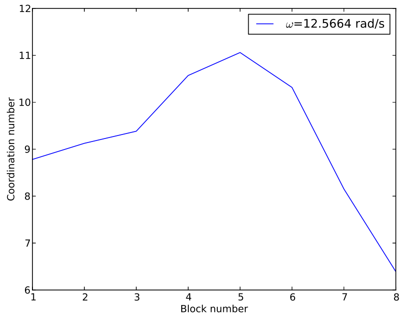
(a) $A = 7\ mm$



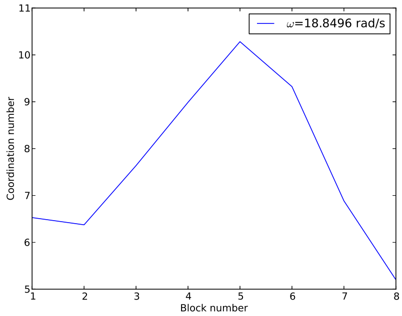
(b) $A = 15\ mm$



(c) $A = 7\ mm$



(d) $A = 15\ mm$



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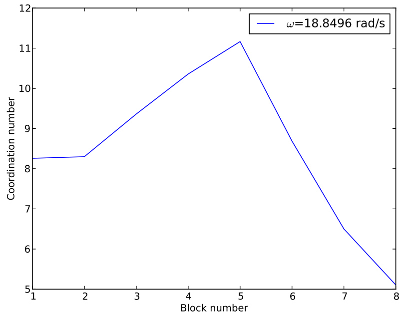

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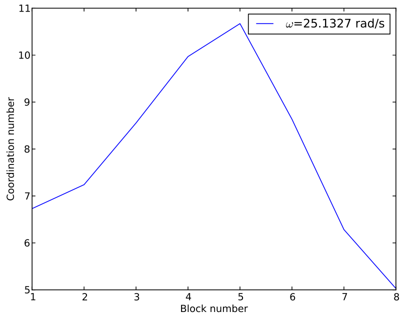
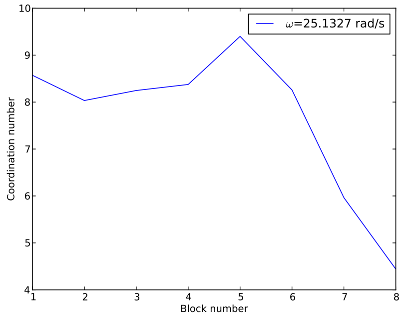


(g) $A = 7\ mm$



(h) $A = 15\ mm$

Figure 10: Coordination number of each block of the assembly for constant displacement amplitude A.

5 Conclusion

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