Dynamic Sensitivity of a Multi-block Stack Subjected to Horizontal Harmonic Excitation

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Abstract :

Dynamic sensitivity of a one-dimensional stack of four rigid blocks with controlled initial gaps, undergoing external harmonic vibrations is investigated. Time variation of the mass inertia and the relative kinetic energy are considered as dynamic block stack attributes. Numerical simulation is based on the Non Smooth Contact Dynamics (NSCD) time integration framework Solfec (http://code.google.com/p/solfec/). Sensitivity space parameters include a range of excitation frequencies and velocity amplitudes.

Key words: block dynamics, Non Smooth Contact Dynamic, self organisation, simulation, energy distribution

1 Introduction

Many civil engineering structures can be considered as discrete, discontinuous systems with deliberate gaps or clearances ranging from simple dry stone walling or masonry to sophisticated complex geometry graphite cores in nuclear power plants. In the latter case, such structural elements are vital safety critical components and there is a need of predict their behaviour under both static and dynamic loadings. In spite of extraordinary advances in nonlinear computational mechanics, it is still difficult to define the highly nonlinear mechanical response of such systems even if some homogenisation technique allows relevant dynamic characterisation.
Indeed dynamic characterisation of such discontinuous structures is complex due to the free travel and gaps as well as quantities such as local deformation or energy transfer between blocks. The simplest way to explore and illustrate relevant observations is to study a one-dimensional model *i.e.* a row comprising N rigid blocks driven by harmonic excitation of the side boundaries, where the blocks are subject to dissipative collisions.

In this contribution we study the behaviour of a stack of four rigid blocks subject to external harmonic excitation due to collisions with a rigid boundary undergoing prescribed velocity history. Experiments and simulations on the column of beads have shown many interesting aspects, such as clustered states and fluidized regime [1,2]. These phenomena depend on the local dissipative mechanical interactions, on the initial configuration of the stack (chosen gap between the blocks)

and also on the frequencies and amplitude of the external excitation. Moreover, the intention is to focus on the possible self organisation of the clustered blocks into oscillatory patterns, associated with distinct frequencies. The control parameters for the study are frequencies and velocity amplitude of the boundary.
Details of the numerical model adopted are given first, followed by the study of the influence of varying excitation frequencies and velocity amplitudes.

2 Numerical Model

2.1 System and simulation

A system of four identical rigid blocks is considered. With the length $s = 36\ mm$ and the section $12 * 12\ mm^2$ (mass density $2700\ kg/m^3$, hence block masses equal to $0.014\ kg$) the blocks are initially configured with a gap g between them within a horizontal box of width $12.5\ mm$ and the total length $L = 4s + 4g$. The first block is initially in contact with the side boundary of the cell and the gap is equal to $g = 6\ mm$. The cell is driven horizontally by a series of sinusoidal motions with amplitude $A_0 = 12\ mm$ and, with frequencies f (angular frequencies $\omega = 2\pi f$) hence the boundary velocity at the time t is defined as

$$V_{cell}(t) = A_0 \omega cos(\omega t)$$

with $A = A_0\omega$ the normalised velocity amplitude in order to achieve constant velocity amplitude for tests at different frequencies. A schematic drawing is given in Figure 1. As this is a one dimensional problem, the friction between blocks and with the cell boundary is neglected $\mu = 0$ and the only dissipation mechanism is the restitution coefficient e for a collision between blocks, the value of $e = 0.9$ is adopted.

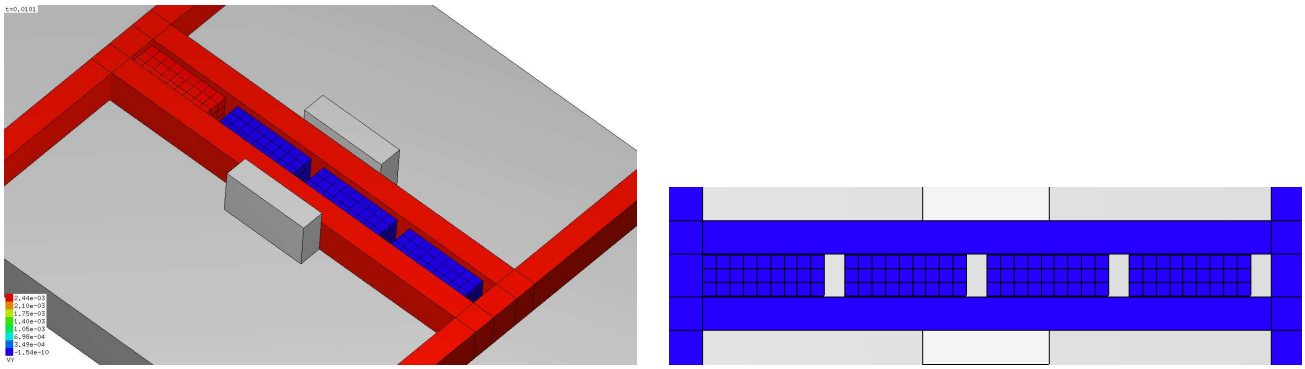


Figure 1: Numerical model

Discrete numerical simulations were performed using the Non Smooth Contact Dynamics (NSCD) method [5], which is specially convenient for rigid blocks. This method is based on an implicit time integration of the equations of motion expressed in terms of velocities and considering generalized nonsmooth contact laws describing noninterpenetration and dry friction between rigid blocks. This formulation unifies the description of lasting contacts and collisions through the concept of an impulse, which can be defined as the time integral of the contact force. The NSCD simulation was applied using the Solfec platform [4].

2.2 Dynamic sensitivity parameters

Dynamic sensitivity of the four block stack under harmonic excitation is studied by extracting time histories of the two scalar parameters from the simulation results – the mass inertia and the relative kinetic energy of the blocks system, which are conveniently captured from the NSCD/DEM method [2,3]. Trajectories of both the left and the right sides of each block as well as the cell boundary are traced, allowing visualisation of shocks between the blocks. For a series of simulations with differing excitation frequencies, the maxima of the relative kinetic energy E_{max} achieved by the system (comprising block i of mass m_i, of position $Y_i(t)$ and velocity $V_i(t)$) is recorded for a given boundary velocity amplitude and a given angular frequency:

$$E_{max} = \left[\sum_i m_i (V_i(t) - V_{cm}(t))^2 \right]_{max}$$

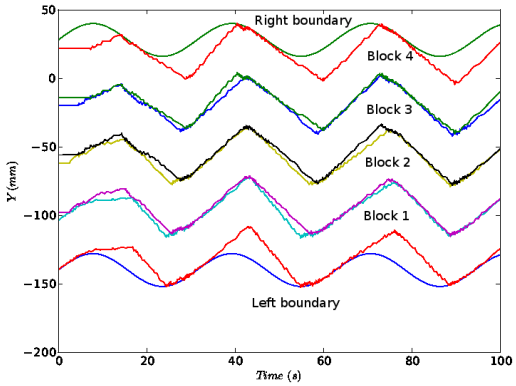
where $V_{cm}(t)$ is the velocity of the block system centroid. In addition, the maximum nondimensional recorded mass inertia index during the excitation is extracted with the mass inertia $I_z(t)$ defined as follows:

$$I_z(t) = \frac{\left[\sum_i m_i (Y_i(t) - Y_{cm}(t))^2 \right]}{I_{ref}}$$

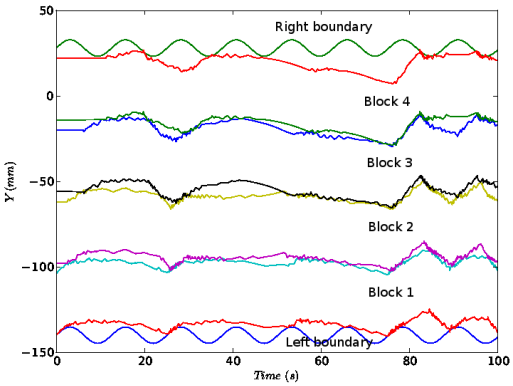
and normalised with respect to the minimum mass inertia I_{ref} the block group has when they are all stuck together. $Y_{cm}(t) = \frac{1}{4} \sum_{i=1}^{4} Y_i(t)$ is the trajectory of the center of mass of the stack.
Finally oscillatory behaviour of the stack can be illustrated with a phase plane diagram by comprising the mass inertia index I_z *vs* the time rate of change of the mass inertia index $\frac{dI_z}{dt}$.

3 Constant velocity amplitude

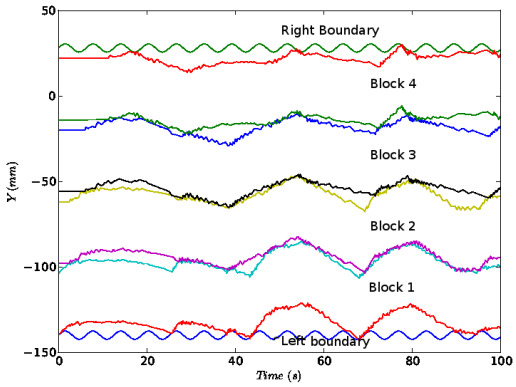
Figure 2 illustrates trajectories (time evolution) of the cell boundary and of the left and the right edges of the four blocks. With the angular frequency $\omega = 0.2$ rad/s the four blocks move almost as a single solid in phase with the boundary (Figure 2(a)). When the excitation frequency ω increases to 0.5 rad/s (Figure 2(b)), the blocks move roughly together however not in phase with the boundary movements. In Figure 2(d), $\omega = 2.0$ rad/s a short transition is detected: from between $t \approx 25$ to $t \approx 50$ seconds, where the stack appears to have split into two parts with two pairs of blocks each moving in opposition phase. With $\omega = 1.0$, $\omega = 3.0$ and $\omega = 10.0$ rad/s Figure 2(c,e,f) each of the blocks moves independently all the time, experiencing multiple collisions.



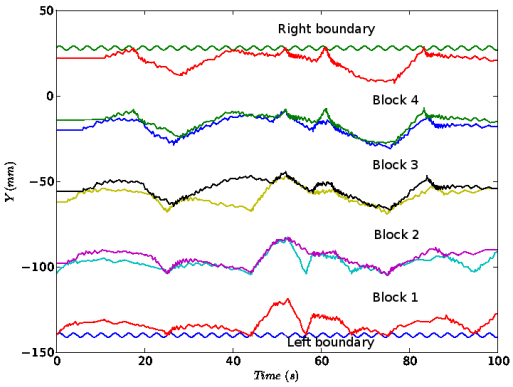
(a) $\omega = 0.2$ rad/s
$A = A_0\omega = 2.4$ mm/s, $V(t) = Acos(\omega t)$



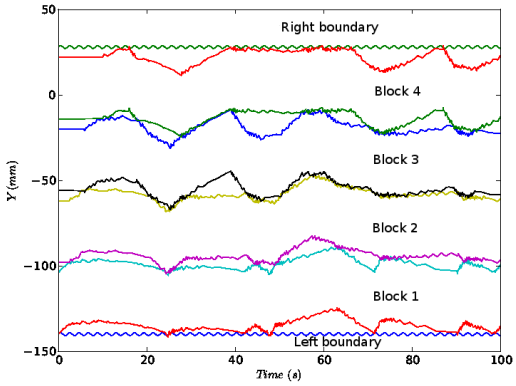
(b) $\omega = 0.5$ rad/s



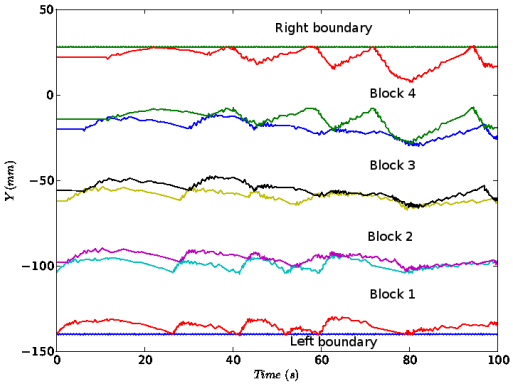
(c) $\omega = 1.0$ rad/s



(d) $\omega = 2.0$ rad/s



(e) $\omega = 3.0$ rad/s

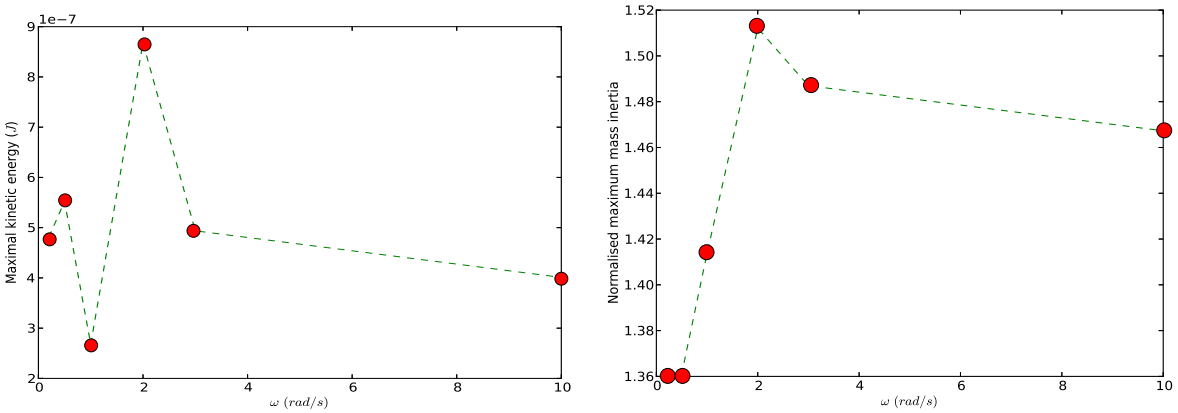


(f) $\omega = 10.0$ rad/s

Figure 2: Trajectories time evolution Y of the left and the right sides of each block and cell, with initial gap $g = 6\ mm$ for constant velocity amplitude $A = 2.4$ mm/s and excitation frequencies ω (a) 0.2 rad/s, (b) 0.5 rad/s, (c) 1.0 rad/s, (d) 2.0 rad/s, (e) 3.0 rad/s and (f) 10.0 rad/s.

Figure 3 illustrates the maximum relative kinetic energy over the entire simulation time achieved by the block system as a function of angular frequencies, for constant velocity amplitude $A = 2.4$ mm/s as before. Two peaks are noted: a small one at $\omega = 0.5$ rad/s and a large one at $\omega = 2.0$ rad/s. It can be observed that there are less collisions between blocks with these angular frequencies, hence there is less dissipation through collisions and consequently the stack of blocks can achieve greater kinetic energy. Similarly to earlier illustrations, the maximum mass inertia over the entire simulation

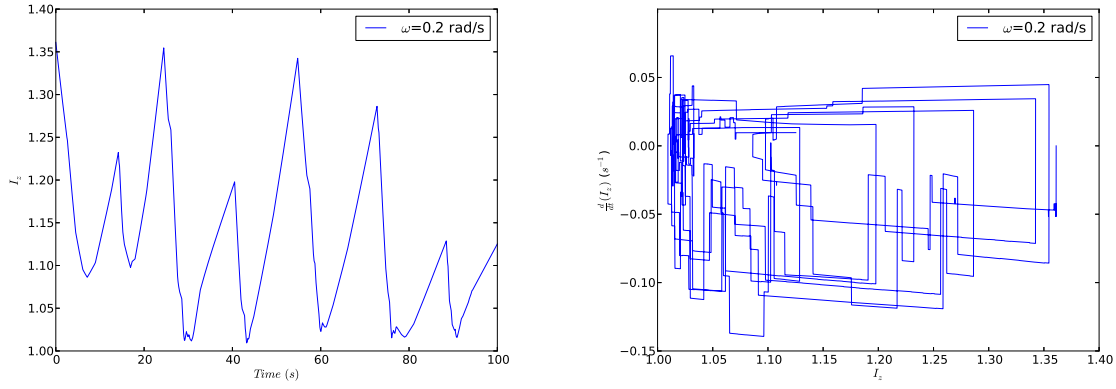
time as a function of the excitation angular frequency is plotted Figure 3(b). The curves have roughly the same shape with the same peak at $\omega = 2.0$ rad/s.



(a) (b)

Figure 3: (a) Maximum relative kinetic energy and (b) maximum mass inertia achieved by the block system according to excitation frequency ω with initial gap between blocks $g = 6\ mm$ for velocity amplitude $A = 2.4\ mm/s$.

Figure 4(a) shows inertia time evolution of the stack, so one can follow how the blocks are distributed within the constrained cell during the simulation for a particular case of $\omega = 0.2$ rad/s. Phase plane diagram of the mass inertia index is plotted in Figure 4(b) and the blocks system doesn't seem to tend towards a steady state response. It should be noted that the problem considered is associated with small gaps and very little freedom for the blocks to move, hence a larger cell size may allow steady state oscillatory responses to develop.

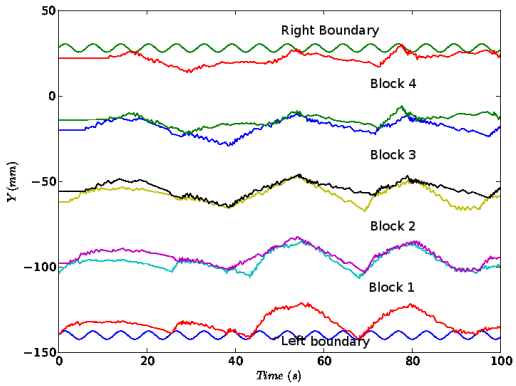


(a) (b)

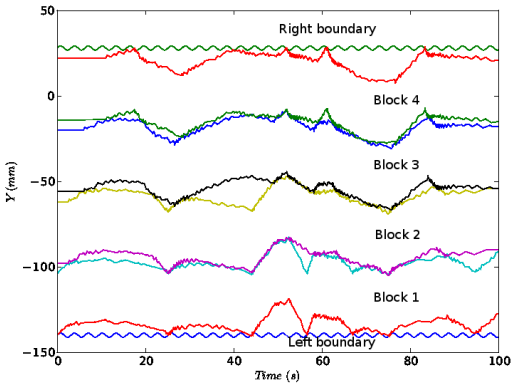
Figure 4: (a) Mass inertia index time evolution I_z and (b) phase plane diagram of the mass inertia index of the stack with initial gap $g = 6\ mm$ for constant velocity amplitude $A = 2.4$ mm/s and excitation frequency $\omega = 0.2$ rad/s.

4 Constant excitation frequency, changing velocity amplitude

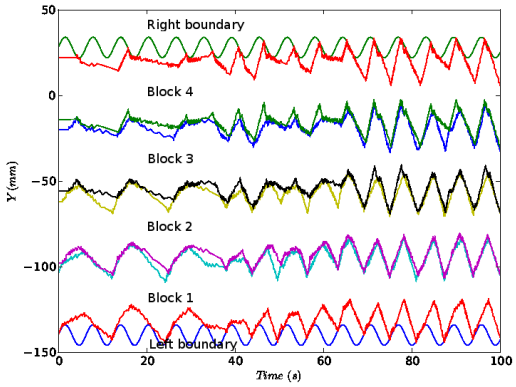
In the next problem, the influence of the changes in the amplitude for a given constant frequency is considered.



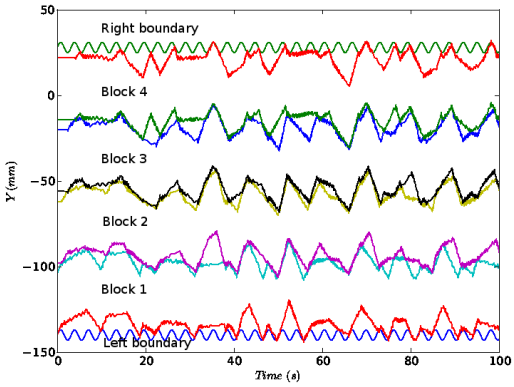
(a) $\omega = 1.0$ rad/s, $A = 2.4$ mm/s



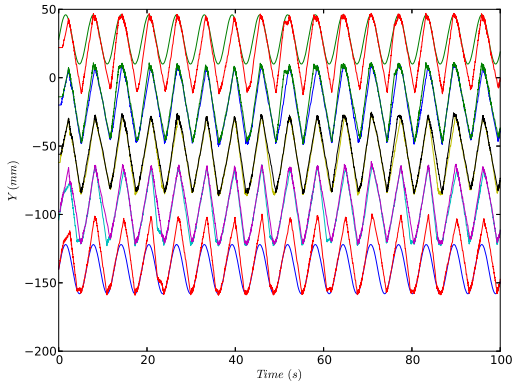
(b) $\omega = 2.0$ rad/s, $A = 2.4$ mm/s



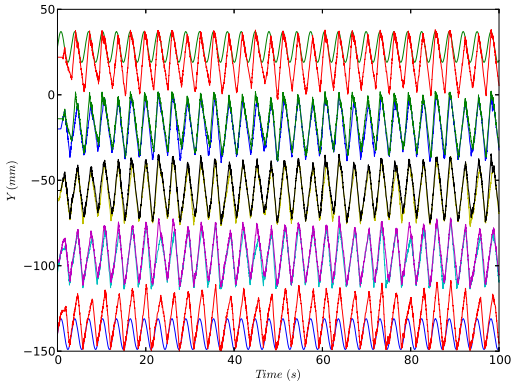
(c) $\omega = 1.0$ rad/s, $A = 6.0$ mm/s



(d) $\omega = 2.0$ rad/s, $A = 6.0$ mm/s



(e) $\omega = 1.0$ rad/s, $A = 18.0$ mm/s



(f) $\omega = 2.0$ rad/s, $A = 18.0$ mm/s

Figure 5: Trajectories time evolution Y of the left and the right sides of each block and cell, with an initial gap $g = 6\ mm$ for constant excitation frequency $\omega = 1.0$ rad/s and velocity amplitude (a) $A = 2.4$ mm/s, (c) $A = 6.0$ mm/s, (e) $A = 18.0$ mm/s ; for constant excitation frequency $\omega = 2.0$ rad/s (b) $A = 2.4$ mm/s (d) $A = 6.0$ mm/s and (f) $A = 18.0$ mm/s.

Figure 5 shows time evolution (trajectories) of the cell boundaries and the left and the right edges of the four blocks for fixed constant excitation frequencies $\omega = 1.0$ rad/s (a,c,e) and $\omega = 2.0$ rad/s (b,d,f). In both cases if the velocity amplitude is increased, the oscillation amplitude increases and the stack tends to move as a single block assembly. In both cases the initial gap between blocks is $g = 6$ mm.

5 Conclusion

A horizontal row of four rigid blocks within a constrained cell has been excited with a harmonic movement of the boundary. Gap between the blocks lead to frequent collisions with one another. The system is dissipative with a restitution coefficient lower than 1. Despite its simplicity, the dynamic response of this system is found to be complex. With the constant velocity amplitude, the system exhibited several response regimes as the angular frequency increased, from a solid like to chaotic displacements within the stack. It was found that extracting maximum relative kinetic energy as a function of the angular excitation frequency one could detect a more organised regime, as the oscillatory behaviour appears less dissipative and associated with less collisions. Simulations with a constant angular frequency and different velocity amplitude has also been considered. The blocks system seems to become more organised with more external energy input.

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