Characteristic lengths in granular piles exhibiting steady surface flows

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ABSTRACT: We investigate numerically granular piles exhibiting steady surface flows. A vertical monolayer of frictional grains is confined between two vertical sidewalls. Above a critical flowing rate and in agreement with experiments (Taberlet, Richard, Valance, Losert, Pasini, Jenkins, & Delannay 2003), surface flows at inclination larger than the angle of repose appear. Below these surface flows, particles exhibit a very slow creep motion whose velocity decays exponentially with depth (Lemieux & Durian 2000, Komatsu, Inagaki, Nakagawa, & Nasuno 2001, Crassous, Metayer, Richard, & Laroche 2008). Here, we focus on the correlations between the surface flow and the creeping region in the case of steady and fully developed flows. We found that the height of the surface flow and the characteristic decay length of the creeping zone are linked through an affine relation which depends on the micromechanical parameters. Therefore the surface flow and the creeping zone are characterized by only one length.

1 INTRODUCTION

Surface flows of dry granular material over an apparently static bed have recently received great attention. Indeed, contrary to flows down rigid inclines (Berton, Delannay, Richard, Taberlet, & Valance 2003, Bi, Delannay, Richard, & Valance 2006, Delannay, Louge, Richard, Taberlet, & Valance 2007, Kumaran 2008), the particle volume fraction varies through the flow height (Richard, Valance, Métayer, Sanchez, Crassous, Louge, & Delannay 2008). Thus, this relatively simple flow configuration permits the simultaneous observation of several different regimes and is ideal to test existing theories or to inspire new ones which aim is to describe and predict the whole behavior of flowing granular matter (Berzi & Jenkins 2011). Near the free surface, there is a dilute region, in which grains experience mainly binary and instantaneous collisions (Jenkins & Richman 1985). Below this collisional layer, one can observe a dense collisional regime, in which correlated motion seems to play a fundamental role (Pouliquen 2004, Mitarai & Nakanishi 2007, Staron 2008) and, finally, at the bed, grains experience enduring contacts that permit creep with an exponentially decaying velocity profile (Komatsu, Inagaki, Nakagawa, & Nasuno 2001, Crassous, Metayer, Richard, & Laroche 2008). To study such kind of flows, two different experimental setups have been employed. In the first one, particles are continuously fed to the top of a heap with constant mass flow rate (Komatsu, Inagaki, Nakagawa, & Nasuno 2001, Taberlet, Richard, Valance, Losert, Pasini, Jenkins, & Delannay 2003, Taberlet, Richard, Henry, & Delannay 2004, Jop, Forterre, & Pouliquen 2005, Richard, Valance, Métayer, Sanchez, Crassous, Louge, & Delannay 2008). The other one, consists of a partially filled drum which is rotated at a constant angular velocity around a horizontal axis (Orpe & Khakhar 2001, Hill, Gioia, & Tota 2003, Taberlet, Richard, & John Hinch 2006, Félix, Falk, & D'Ortona 2007). In the former, steady and fully developed (i. e. invariant in the main flow direction) flows are obtained, while in the latter, they are steady, but not fully developed. Experimental and numerical results obtained using the two devices indicates that (i) unlike flow over rigid beds (Pouliquen 1999) the angle of inclination of the free surface and the depth of the flow above the bed are fully determined by the mass flow rate over the heap and the angular velocity of the rotating drum, (ii) the presence of frictional sidewalls plays a fundamental role in controlling surface granular flows, permitting steady and fully developed flows at angles of inclination of the free surface much higher than the angle of repose of the granular material and (iii) below the surface flow the grains are not static but move in an intermittent way i.e. they creep. The comprehension of such creeping flow, and its connection with the surface flow, are still debated topics. Komatsu et al. (Komatsu, Inagaki, Nakagawa, & Nasuno 2001) show that the mean velocity of the creep region decays exponentially with depth, and the characteristic decay length is approximately equal to the particle size, d:

$$\langle v(h)\rangle = v_0 \exp(-y/\lambda) \text{ with } \lambda \approx d \quad (1)$$

The exponential character of the vertical velocity profile has been confirmed by a careful study of the creeping motion using Dynamic Light Scatering (Crassous, Metayer, Richard, & Laroche 2008).

A combined experimental and numerical study (Richard, Valance, Métayer, Sanchez, Crassous, Louge, & Delannay 2008) has shown that displacement of the grains in the so-called creeping zone is intermittent even in the case of steady and fully developed flows. Moreover, the probability for a grain to move was found to decays with depth. Interestingly those authors show that the characteristic length of the velocity decay λ is proportional to the height of the continuously flowing layer. Unfortunately, due to experimental limitations, the flow studied are relatively thin and the validity of that result in the case of thick flows is an open question. To address this point, we will study, by means of numerical simulations flows of granular matter down a heap. To obtain flows of several tens of grain size with satisfactory statistics we choose to work with quasi 2D system, i.e. a vertical monolayer of grains confined between two vertical sidewalls.
The papers is organized as follows. The section 2 describes details of the numerical method used. The properties of the flowing region are studied in sections 3. Section 4 is devoted to the study of the creeping zone and, particularly, to the determinination of the characteristic length of the velocity decay. The rheology of the flow, as for it, is studied in section 5. Finally our conclusions are presented in section 6.

2 SIMULATION METHODOLOGY

We use molecular dynamics simulations (MD). Grains are cohesionless spheres of diameter uniformly distributed between $0.8d$ and $1.2d$ and with mass m. The system is spatially periodic in the flow direction with length and width respectively of $25d$ and $1.1d$ (see Figure 1). The cell is bounded by a bumpy bed at the bottom and an open top. Therefore, this monolayer is considered and studied as a quasi two dimensional system. The initial configuration consists of an ordered square array of non-overlapping spheres with random velocities which is subsequently allowed to settle under gravity g on top of the bumpy bottom. The latter is built using the following method: We first use as bottom a gluing flat wall. The grains contacting that wall are then stuck on it and the whole (the wall and the glued grains) form the bumpy bottom. The system is then inclined at the angle θ with respect to the horizontal and the measurements of the flow properties are performed once the flow is steady and fully developed.

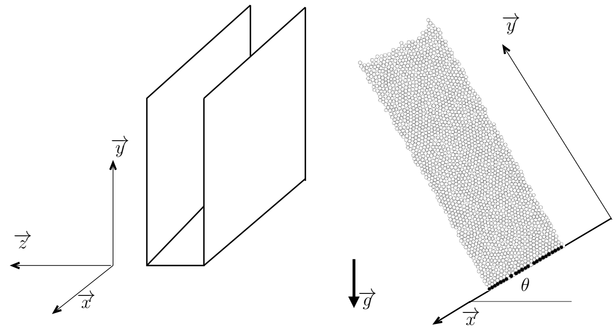

Figure 1: Sketch of the set-up and cartesian coordinate system with unit vector **x** along the flow, normal to the free surface and perpendicular to sidewalls with origin at the bottom of the cell.

We use our own implementation (Richard, Valance, Métayer, Sanchez, Crassous, Louge, & Delannay 2008, Taberlet & Richard 2006) of the classical discrete element method method where Newton's equations of motion for a system of N soft grains are integrated (Walton 1984, Cundall & Strack 1979). This technique is able to reproduce a broad range of experimental results including gravity driven flows (see Ref. Delannay, Louge, Richard, Taberlet, & Valance 2007 and references therein). It requires an explicit expression for the forces that act between two contacting grains. Since this technique is well known, we just present here the forces used in this work. For the normal force between two overlapping grains, $\mathbf{F_n}$ a standard linear spring-dashpot interaction model (Cundall & Strack 1979) is used, $\mathbf{F}_n = k_n \delta \mathbf{n}_{ij} - \gamma^n \mathbf{v}_n$, where δ_n is the normal overlap, $\mathbf{n}_{ij}$ the normal unit vector of the contact, k_n is the spring constant, γ_n the damping coefficient, and $\mathbf{v_n}$ the normal relative velocity. The use of a damping coefficient is necessary to model the dissipation characteristic of granular materials. Likewise the tangential force is modeled as a linear elastic and linear dissipative force in the tangential direction is given by $\mathbf{F}_t = -k_t \Delta \mathbf{s}_{ij} - \gamma^t \mathbf{v}_t$, where k_t is the tangential spring constant, $\Delta \mathbf{s}_{ij}$ the tangential overlap, γ_t the tangential damping, and $\mathbf{v_t}$ the tangential velocity at the contact point. The magnitude of $\Delta \mathbf{s}_{ij}$ is truncated as necessary to satisfy the Coulomb law $|\mathbf{F}_t| \leq \mu_g |\mathbf{F}_n|$, where μ_g is the grain-grain coefficient. Impacts against the sidewalls are treated as collisions with a sphere of infinite mass and radius, which mimics a large flat surface. The sidewall-grain friction coefficient is called μ_w. The presented simulations were carried out with the following set of materials parameters: the normal spring constant $k_n = 5.6 \times 10^6 mg/d$ and the tangential spring constant $k_t = 2k_n/7$ (Shäfer, Dippel, & Wolf 1996). The normal damping coefficient is chosen such as the normal coefficient restitution is $e = 0.88$. The tangential damping is set to zero. The equations are integrated using a velocity-verlet algorithm and a time step $\Delta t = 7 \times 10^{-5}\sqrt{d/g}$.

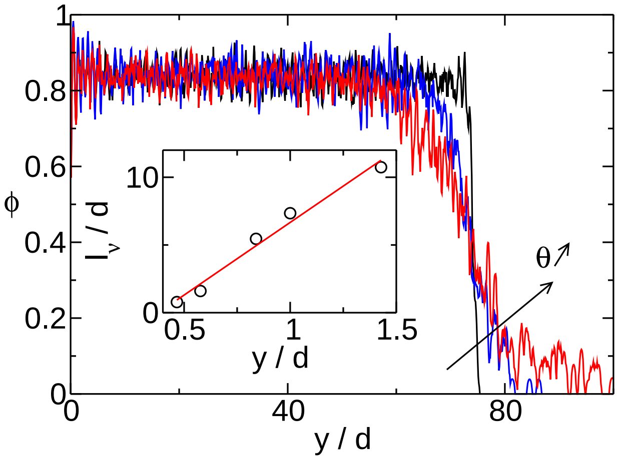

Figure 2: (Color online) Vertical profile of solid fraction ϕ at inclinations shown versus depth y/d (the bottom of the cell is located at 0) for $\mu_w = 0.3$ and $\mu = 0.3$. Inset variations of the scale l_ν with $\tan\theta$. The dashed line corresponds to the best fit.

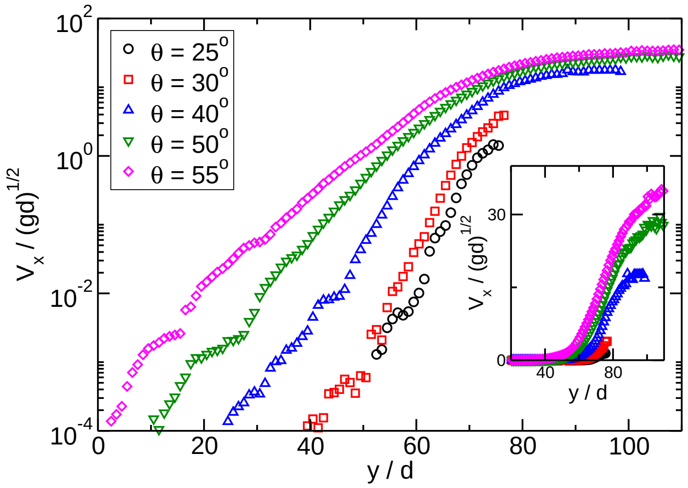

Figure 3: (Color online) Semi-log velocity profiles $V_x/\sqrt{gd}$ at inclinations shown versus depth y/d (inset: linear-linear profiles) for friction coefficient $\mu_g = 0.3$ and $\mu_w = 0.3$.

3 DETERMINATION OF THE HEIGHT OF THE FLOWING REGION

In order to determine the depth of the flowing layer, we follow the method described in (Richard, Valance, Métayer, Sanchez, Crassous, Louge, & Delannay 2008) where that height is obtained from the vertical profiles of the packing fraction. Figure 2 shows a solid fraction profile of the system obtained for $\mu_g = 0.3$ and $\mu_w = 0.3$. From the latter profile, one can distinguish three regions in the depth of the flow. From the bottom: a dense quasistatic bulk where $\phi_0 \approx 0.8$, a flowing layer where solid fraction decreases dramatically and a very dilute zone where spheres experience rare collisions.

These results are similar to the experimental ones (Taberlet, Richard, Valance, Losert, Pasini, Jenkins, & Delannay 2003) and very close to numerical results obtained in 3D (Richard, Valance, Métayer, Sanchez, Crassous, Louge, & Delannay 2008). In the latter work the height h of the flowing layer is defined by fitting the solid fraction profiles using the following expression:

$$\phi(y) = \frac{\phi_0}{2}\left(1 + \tanh\left(-\frac{(y-y_0)}{l_\nu}\right)\right), \qquad (2)$$

where $\phi = \phi_0/2$ for $y = y_0$ and $l_\nu = h/2$ is the characteristic length of the flowing zone. Variations of the length l_ν with $\tan\theta$ is reported on the inset of figure 2. Whatever the values of the grain-grain friction coefficient and of the grain-wall riction coefficient, l_ν increases linearly with $\tan\theta$ which is consistent with previous results (Richard, Valance, Métayer, Sanchez, Crassous, Louge, & Delannay 2008). That linear relation holds for all the simulations we performed but the obtained slopes and intercepts depend on values of friction coefficients (μ_g and μ_w). The study of this effect, which is out of the scope of the present paper, will be adressed elsewhere.

4 DETERMINATION OF THE CHARACTERISTIC LENGTH OF THE VELOCITY DECAY

Velocity profiles for friction coefficients $\mu_g = 0.3$ and $\mu_w = 0.3$ are reported on Figure 3. Similarly to what is observed with the solid fraction profiles, one can define three regions: (i) the top layers where grain motions are clearly ballistic, (ii) the dense flow zone where the velocity is roughly linear and (iii) the creeping zone where the velocity seems to decay exponentially.

Such profiles have been observed in several experimental works (Crassous, Metayer, Richard, & Laroche 2008, Richard, Valance, Métayer, Sanchez, Crassous, Louge, & Delannay 2008). In agreement with the literature (Komatsu, Inagaki, Nakagawa, & Nasuno 2001, Crassous, Metayer, Richard, & Laroche 2008) we also observe that in the creeping zone, the grain velocity decays exponentially as $V_x \propto \exp(y/\lambda)$ where λ is the characteristic length for this exponential decay. In such a zone, the grains move by infrequent, rapid jumps between successive cages (Richard, Valance, Métayer, Sanchez, Crassous, Louge, & Delannay 2008). In the literature (Komatsu, Inagaki, Nakagawa, & Nasuno 2001, Crassous, Metayer, Richard, & Laroche 2008), the authors report a characteristic length of the exponential decay close to the grain size. However, the angles of the flows and thus the sizes of the surface flows were relatively small. On the contrary, here, due to important sidewall friction and flow rate, the angle may reach more than $55°$ and the size of the surface flows can reach several tens of grain size. By fitting the velocity profiles in the creeping zone, we determine the value of λ. Our results, shown on Fig. 4, clearly demonstrate that for important flow angles, the exponential characteristic length can be much larger than the grain size. Interestingly, both the friction coefficients between grains and between walls and grains influence the relation between h and λ although this relation is always affine. Qualitatively, we can observe that

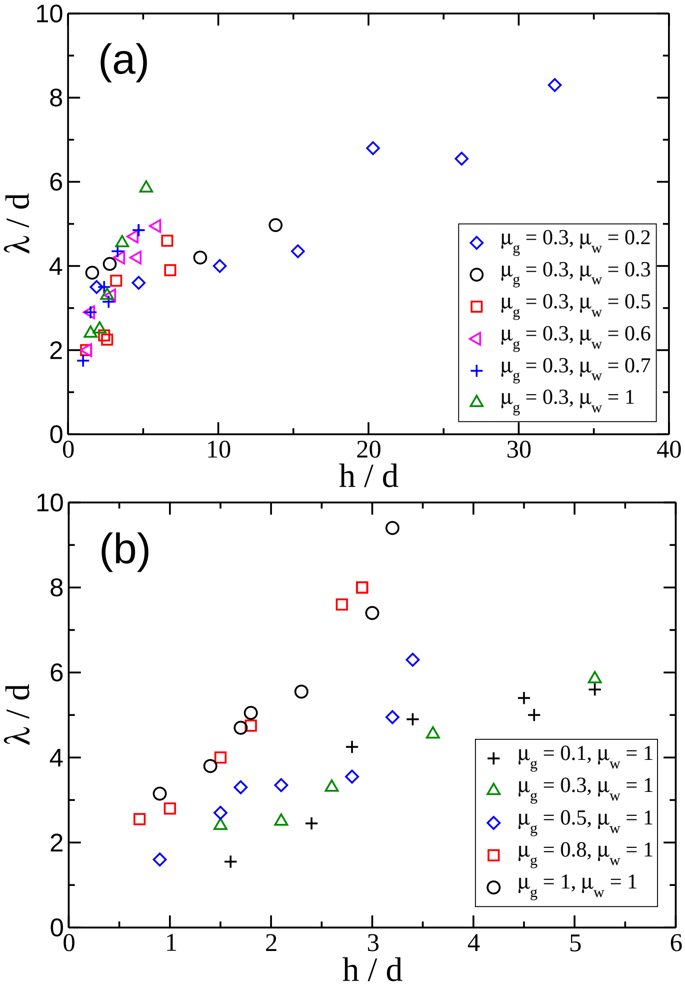

Figure 4: (Color online) Characteristic length of the velocity decay λ versus the height h of the flowing zone with (a) constant friction coefficient $\mu_g = 0.3$ and (b) constant friction coefficient $\mu_w = 1$.

increasing the grain-wall friction coefficient leads to more important values of λ for the same surface flow depth (see Fig. 4a). Increasing the grain-grain friction drives to the same consequence (see Fig. 4b) but the effect is much less important. Unfortunately our statistics are not yet important enough to quantify the effect of those friction coefficients on the linear relation.

Our results not only confirm the experimental finding (Richard, Valance, Métayer, Sanchez, Crassous, Louge, & Delannay 2008) that both the flow depth, h, and the characteristic decay length in the creeping zone λ are proportional but also extend it to thick flows where $\lambda \gg d$. Therefore both the dense flow and creeping zones are characterized by only one length which means that the two zones cannot be treated separately. Clearly any theory which aim is to describe and predict the whole behavior of flowing granular matter should take into account the interactions between those two zones or treat them as a whole.

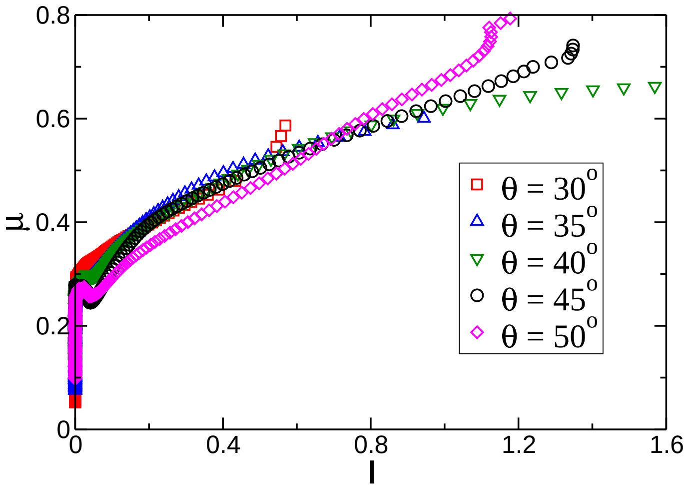

Figure 5: (Color online) The effective coefficient of friction $\mu = \tau/P$ at inclinations shown versus inertial number I for friction coefficients $\mu_g = 0.3$ and $\mu_w = 0.6$.

5 RHEOLOGY

Recently, a purely local rheological description of dense granular flows has been developed successfully (GDR-MiDi 2004, da Cruz, Emam, Prochnow, Roux, & Chevoir 2005). Based on a coulombic friction model it relates the value of the effective coefficient of friction μ (i.e. the ratio of tangential stress τ to normal stress P) to a non-dimensional number I, called the inertial number, which compares the typical time scale of microscopic rearrangements with the typical time scale of macroscopic deformations: $\mu = \tau/P$ and $I = \dot{\gamma} d/\sqrt{P/\rho_s}$, where ρ_s is the grain density and $\dot{\gamma}$ the shear rate. Note that the so-called inertial number I is the square root of the Savage number (Savage 1984) also called the Coulomb number (Ancey, Coussot, & Evesque 1999). It has been empirically shown (Jop, Forterre, & Pouliquen 2005) that, in the case of dense granular flows, the effective coefficient of friction μ of the system can be expressed by the following expression:

$$\mu(I) = \mu_s + \frac{\mu_2 - \mu_s}{I_0/I + 1}.$$

In the latter expression, μ_s corresponds to the angle of repose of the material, i.e. the angle obtained when approaching the quasistatic regime. Consequently, the granular material flows only if the yield criterion $\tau > \mu_s P$ is satisfied. In strongly sheared regimes ($I \gg 1$), $\mu(I)$ grows asymptotically towards μ_2. It should be pointed out that the $\mu(I)$ rheology should be applied only to monodirectional flows. Indeed the extension to the $3D$ case is not straightforward since stress and strain tensors are not always aligned (Cortet, Bonamy, Daviaud, Dauchot, Dubrulle, & Renouf 2009, Brodu, Richard, & Delannay 2013). Our system is strongly confined by the two sidewalls between which the grains flow, so one may wonder if the $\mu(I)$ rheology is still valid in that case. Figure 5 reports curves $\mu(I)$ for several angles and for $\mu = 0.3$ and $\mu_w = 0.6$. Let us point out that we report only the points that correspond to the dense flow zone and to the creeping zone. We remove those corresponding to the gaseous layer where the $\mu(I)$ rheology is not relevant. Although the points corresponding to the creeping zone have not been removed, it is not relevant to apply the $\mu(I)$ rheology in that zone, where it predicts that the system does not flow. It can be shown that the $\mu(I)$ rheology remains valid for angles $\theta \leq 45°$. The only differences between the curves correspond to large I, thus to regimes close to the gaseous one. For $\theta = 50°$ significant differences are visible. This is probably due to the fact that for such an important angle, the dense regime is not clearly obtained.

Since our flow is mainly unidirectional the $\mu(I)$ rheology holds in the dense flow zone, an only in that zone, as long as the solid fraction is large enough. In order to captures the dense flow and creeping zones long range correlations should be taken into account. A possible way to overcome this flaw, consists in introducing non-local effects (see e.g. (Pouliquen & Forterre 2009, Kamrin & Koval 2012)). Another questionable point is that such a rheology does not use the notion of granular temperature which is at the base of the kinetic theory (Jenkins & Richman 1985) even in the case of dense flows. This may explain the discrepancy observed in Fig. 5 for the largest angle.

6 CONCLUSIONS

In summary, we have numerically studied granular piles exhibiting steady and fully developed surface flow. Our system is a vertical monolayer of grains confined between two sidewalls. As expected, we obtained a rapid surface flow –characterized by its depth h– and, below this flow, a slow creep motion in which the grain velocity decays exponentially with a characteristic length λ. Contrary to previous studies we show that the latter length can of the order of several tens of grain sizes for large flow angles. We also confirm than h and λ are related through and affine law whose parameters depend on grain-grain and grain-wall friction coefficients.

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